CH # 11 HEAT

Some important formulae:

i) $\alpha = \frac{\Delta L}{L_1 \Delta T}$ ii) $L_2 = L_1(1 + \alpha\Delta T)$

iii) $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ iv) $P = \rho gh\ (liquid)$

iv) $\Delta Q = mc\Delta T$ (specific heat) v) $\Delta Q = c\Delta T$(heat capacity) vi) $\Delta Q = mL$ (Latent heat)

vii) Heat lost = Heat gain (law of heat exchange)

viii) $\text{Power} = \frac{Total\ energy}{Time\ taken}$ ix) $\Delta Q = \frac{KA\Delta T\Delta t}{\Delta L}$

11.1: A metal rod of diameter 1cm measures 50cm in length at 20^{0}C. When it is heated to 95^{0}C, its length becomes 50.06cm. What is the coefficient of linear expansion of the rod? What will be its length when it is cooled to 0^{0}C?

GIVEN:

Length of rod at 20^{0}C = L_1 = 50cm
Length of rod at 95^{0}C = L_2 = 50.06cm
Change in length = ΔL = 50.06 – 50 = 0.06cm
Initial temperature = T_1 = 20^{0}C
Final temperature = T_2 = 95^{0}C
Change in temperature = ΔT = 95 – 20 = 75^{0}C

REQUIRED:

Coefficient of linear expansion of the rod = α = ?

SOLUTION:

$$\alpha = \frac{\Delta L}{L_1 \Delta T}$$
$$\alpha = \frac{0.06}{(50)(75)}$$
$$\alpha = \frac{0.06}{3750}$$
$$\alpha = 0.000016\ ^0C^{-1}$$
$$\alpha = 16 \times 10^{-6}\ ^0C^{-1}$$

Now we have to find length when it is cooled to 0^{0}C,
$L_2 = L_1(1 + \alpha\ \Delta T)$
Where the temperature changes from 20^{0}C to 0^{0}C so,
ΔL = 0 –20 = – 20^{0}C
Now,
$L_2 = 50\{1 + 0.000016 \times (-20)\}$
$L_2 = 50(1 - 0.000016 \times 20)$
$L_2 = 50(0.99968)$
L_2 = 49.984cm

11.2: The difference between length of two rods A and B is 60cm at all temperature. Find their original lengths at 0^{0}C. Given $\alpha_A = 18 \times 10^{-6}\ ^0C^{-1}$ and $\alpha_B = 27 \times 10^{-6}\ ^0C^{-1}$.

GIVEN:

Coefficient of linear thermal expansion of rod A = α_A = $18 \times 10^{-6}\ ^0C^{-1}$
Coefficient of linear thermal expansion of rod B = α_B = $27 \times 10^{-6}\ ^0C^{-1}$

REQUIRED:

Original length of rod A = L_A
Original length of rod B = L_B

SOLUTION:

According to given condition,
$\Delta L_A = \Delta L_B$
$\alpha_A L_A \Delta T = \alpha_B L_B \Delta T$

$\alpha_A L_A = \alpha_B L_B$

$$\frac{L_A}{L_B} = \frac{\alpha_A}{\alpha_B} - - - - - - - -(1)$$

Where

$\Delta L = L_A - L_B$

$60 = L_A - L_B$

$L_B = L_A - 60 - - - - - - - -(2)$

Put it in (1),

$$\frac{L_A}{L_A - 60} = \frac{18 \times 10^{-6}}{27 \times 10^{-6}}$$

$$\frac{L_A}{L_A - 60} = \frac{18}{27}$$

$$\frac{L_A}{L_A - 60} = \frac{3}{2}$$

$2L_A = 3(L_A - 60)$

$2L_A = 3L_A - 180$

$3L_A - 2L_A = 180$

$L_A = 180cm$

Put it in equation (2),

$L_B = 180 - 60$

$L_B = 120cm$

11.3: WA gas of a given mass at a pressure of 50cm of Hg is heated from 27^0C to 97^0C. If the volume is maintained constant, calculate the pressure exerted by the gas.

GIVEN:

Pressure = P_1 = 50cm of Hg
Temperature = $T_1 = 27^0C = 27 + 273 = 300K$
Temperature = $T_2 = 97^0C = 97 + 273 = 370K$

REQUIRED:

Pressure = P_2 = ?

SOLUTION:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Since pressure is constant,

$V_1 = V_2 = V$

$$\frac{P_1V}{T_1} = \frac{P_2V}{T_2}$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$P_2 = \frac{P_1T_2}{T_1}$$

$$P_2 = \frac{50 \times 370}{300}$$

P_2 = 61.67 cm of Hg

11.4: If a given mass of a gas has a volume of 4.5 x $10^{-5}m^3$ at a pressure of 30.0KPa what will be the volume of the gas if the pressure is increased to 50.0KPa while the temperature is kept constant?

GIVEN:

Volume of gas at 30.0KPa = $V_1 = 4.5 \times 10^{-5}m^3$
Pressure = P_1 = 30.0KPa

Pressure = P_2 = 50.0KPa

REQUIRED:

Volume of gas at 50.0KPa = V_2 = ?

SOLUTION:

According to Boyle's law,

$P_1V_1 = P_2V_2$

$$V_2 = \frac{P_1V_1}{P_2}$$

$$V_2 = \frac{30 \times 4.5 \times 10^{-5}}{50}$$

$V_2 = 2.7 \times 10^{-5}m^3$

11.5: A bubble of air rises from the bottom of a pond to the surface. Just before the bubble reaches the surface it breaks and the volume was double its original volume. The water has a uniform temperature and its density is $1000Kgm^{-3}$. Atmospheric pressure is 10^5Pa. Estimate the depth of the pond.

GIVEN:

Density of water = $\rho_1 = 1000Kgm^{-3}$
Pressure = P = 10^5 Pa

REQUIRED:

Depth of pond = d = ?

SOLUTION:

$P = \rho g d$

$$d = \frac{P}{\rho g}$$

$$d = \frac{10^5}{1000 \times 9.8}$$

d = 10.2m

11.6: An electric heater of power 600W raises the temperature of 4.0kg of a liquid from 10^0C to 15^0C in 100Sec. Calculate,
i) The heat capacity of the 4.0Kg of the liquid.
ii) The specific heat capacity of the liquid.

GIVEN:

Power of electric heater = P = 600W
Time taken = t = 100Sec
Mass of liquid = m = 4.0Kg
Temperature = $T_1 = 10^0C = 10 + 273 = 283K$
Temperature = $T_2 = 15^0C = 15 + 273 = 288K$
Change in temperature = $\Delta T = 288 - 283 = 5K$

REQUIRED:

(i) Heat capacity = c = ?

(ii) Specific heat capacity = C = ?

SOLUTION:

$$\text{(i) } C = \frac{\Delta Q}{\Delta T}$$

As we know that ΔQ is the amount of energy,

ΔQ = Power x time

ΔQ = 600 x 100

ΔQ = 60,000 J

Now,

$$C = \frac{60{,}000}{5}$$

C = 12000J/K

$$\text{ii) } C = \frac{\Delta Q}{m\Delta T}$$

$$C = \frac{60{,}000}{4 \times 5}$$

C = 3000 J/Kg.K

11.7: A 2KW steel kettle of mass 1Kg contains 1.5Kg of water at 30^{0}C. What is the time taken to boil the water if the specific heat capacity of steel is 460JKg^{-1} ^{0}C^{-1} and that of water is 4200 JKg^{-1} ^{0}C^{-1}?

GIVEN:

Power of kettle = P = 2Kw = 2000W

Mass of kettle(Steel) = 1Kg

Mass of water = m = 1.5Kg

Temperature of water = 30^{0}C

Boiling temperature of water = 100^{0}C

Change in temperature = ΔT = 100 – 30 = 70^{0}C

Specific heat capacity of steel = C = 460 J/Kg.^{0}C

Specific heat capacity of Water= C = 4200 J/Kg.^{0}C

REQUIRED:

Time taken to boil water = t = ?

SOLUTION:

As we know that,

$$Power = \frac{Total\ energy}{Time}$$

$$Time = \frac{Total\ energy}{Power} - - - - - - - - (1)$$

Where,

ΔQ = mCΔT

For water,

$\Delta Q_{(Water)}$ = 1.5 x 4200 x 70

ΔQ = 441000 J

For Steel,

$\Delta Q_{(Steel)}$ = 1 x 460 x 70

ΔQ = 32200 J

Total energy = $\Delta Q_{(Steel)} + \Delta Q_{(Water)}$

Total energy = 32200 + 441000

Total energy = 473200

Put it in equation (1),

$$Time = \frac{473200}{2000}$$

Time = 236.6 Sec or approx. 237 sec

11.8: 1 liter of water at 100^{0}C is added to 5 liters of water at 30^{0}C. What is the final temperature of water?

GIVEN:

Temperature of 1 liter water = T_1 = 100^{0}C

Temperature of 5 liter water = T_2 = 30^{0}C

Specific heat of 1 liter water = C_1 = 4200 J/Kg.^{0}C

Specific heat of 5 liter water = C_2 = 4200 J/Kg.^{0}C

REQUIRED:

Final temperature = T = ?

SOLUTION:

Heat lost by 1 liter water:

$\Delta Q_1 = m_1C_1\Delta T$

ΔQ_1 = 1 x 4200 x (100 – T)

ΔQ_1 = 420000 – 4200T

Heat gained by 5 liter water:

$\Delta Q_2 = m_2C_2\Delta T$

ΔQ_2 = 5 x 4200 (T – 30)

ΔQ_2 = 21000 (T – 30)

ΔQ_2 = 21000T – 630000

According to law of heat exchange,

Heat lost by hot body = Heat gained by cooled body

$$\Delta Q_1 = \Delta Q_2$$

420000 – 4200T = 21000T – 630000

420000 + 630000 = 21000T + 4200T

1050000 = 25200T

$$T = \frac{1050000}{25200}$$

T = 41.67^{0}C

11.9: Two liquids A and B are at temperatures 60^{0}C and 20^{0}C respectively. Their masses are in the ratio 3:4 and their specific heats are in the ratio 4:5. Calculate the final temperature of mixture if the liquids A and B are mixed. Neglect the water equivalent of the calorimeter.

GIVEN:

Liquid A:

Temperature = 60^{0}C

Mass = 3

Specific heat = 4

$\Delta Q_1 = m_1 C_1 \Delta T$

$\Delta Q_1 = 3 \times 4 \times (60 - T)$

$\Delta Q_1 = 12 \times (60 - T)$

$\Delta Q_1 = 720 - 12T$

Liquid B:

Temperature = 20^{0}C

Mass = 4

Specific heat = 5

$\Delta Q_2 = m_2 C_2 \Delta T$

$\Delta Q_2 = 4 \times 5 \times (T - 20)$

$\Delta Q_2 = 20 \times (T - 20)$

$\Delta Q_2 = 20T - 400$

REQUIRED:

Final temperature = T = ?

SOLUTION:

According to law of heat exchange,

Heat lost by hot body = Heat gained by cooled body

$$\Delta Q_1 = \Delta Q_2$$

720 – 12T = 20T – 400

720 + 400 = 20T + 12T

1120 = 32T

$$T = \frac{1120}{32}$$

T = 35^{0}C

11.10: 1Kg of water is contained in a 1.25Kilowatt kettle. Assuming that the heat capacity of the kettle is negligible, calculate the time taken for the temperature of water to rise from 25^{0}C to its boiling point at 100^{0}C.

GIVEN:

Power of kettle = P = 1.25Kw = 1250Watt

Mass of water = m = 1Kg

Temperature rises from 25^{0}C to 100^{0}C

Change in temperature = ΔT = 100 – 25 = 75^{0}C

Specific heat of water = C = 4200 J/Kg.^{0}C

REQUIRED:

Time taken = t = ?

SOLUTION:

As we know that,

$$Power = \frac{Total\ energy}{Time}$$

$$Time = \frac{Total\ energy}{Power} - - - - - - - - (1)$$

Where,

$\Delta Q = mC\Delta T$

For water,

$\Delta Q = 1 \times 4200 \times 75$

ΔQ = 315000 J

Put it in equation (1),

$$Time = \frac{315000}{1250}$$

Time = 525 Sec which is equal to 4 min and 12 sec.

11.11: 20gram of ice at 10^{0}C is converted into steam at 100^{0}C. Find out the total heat energy required to accomplish the change. Given the specific heat of ice, latent heat of ice and latent heat of steam as 2.1 J/g, 336 J/g and 2268 J/g respectively.

GIVEN:

Mass of ice = 20gm

Temperature of ice = T_1 = -10^{0}C

Temperature of steam = T_2 = 100^{0}C

Specific heat of ice = C_1 = 2.1 J/g.K

Specific heat of water = C_2 = 4.2 J/g.K

Latent heat of ice = L_F = 336 J/g

Latent heat of steam = L_V = 2268 J/g

Rise in temperature = ΔT = 100^{0}C

REQUIRED:

Total energy required to accomplished the change = Q = ?

SOLUTION:

Heat supplied to ice -10^{0}C is utilized to change the ice at -10^{0}C

The following changes take place:

i) Changing of ice from -10^0C to 0^0C.
ii) Changing of ice from 0^0C to 0^0C.
iii) Changing of ice from 0^0C to 100^0C.
iv) Changing of ice from 100^0C to 100^0C.

1) Heat taken by ice from -10^0C to $0^0C = Q_1$

Q_1 = mass x specific heat x rise in temperature

$Q_1 = 20 \times 2.1 \times 10$

$Q_1 = 420$ J

2) Heat taken by ice from 0^0C to $0^0C = Q_2$

Q_2 = mass x Latent heat of ice

$Q_2 = 20 \times 336$

$Q_2 = 6720$ J

3) Heat taken by water from 0^0C to $100^0C = Q_3$

Q_3 = mass x specific heat x rise in temperature

$Q_3 = 20 \times 4.2 \times 100$

$Q_3 = 8400$ J

4) Heat taken by steam from 100^0C to 100^0C Steam = Q_4

Q_4 = mass x latent heat of steam

$Q_4 = 20 \times 2268$

$Q_4 = 45360$ J

Total heat taken by the ice from -10^0C to $100^0C = Q$

$Q = Q_1 + Q_2 + Q_3 + Q_4$

$Q = 420 + 6720 + 8400 + 45360$

Q = 60900 J is the total heat required to accomplished the change.

11.12: Two vessels made of different metals are similar in shape and size. They are fully filled with ice at 0^0C.By the heat from outside all the ice in one vessel melts in 25minutes and that in the other vessel in 20minutes. Compare their conductivities.

GIVEN:

Time interval for First vessel = Δt_1 = 25min

Time interval for second vessel = Δt_2 = 20min

REQUIRED:

$$\text{Comparison of conductivity} = \frac{K_1}{K_2} = ?$$

SOLUTION:

Let K_1 and K_2 be the conductivity of the first and the second vessel.

$\Delta T_1 = \Delta T_2 = \Delta T$ (Change in temperature)

$\Delta L_1 = \Delta L_2 = \Delta L$ (Thickness)

$A_1 = A_2 = A$ Area

We know that,

$$\Delta Q = \frac{KA\Delta T\Delta t}{\Delta L}$$

For vessel No.1 and 2,

$$\Delta Q = \frac{K_1 A\Delta T \times 25}{\Delta L} - - - - - - - - (1)$$

$$\Delta Q = \frac{K_2 A\Delta T \times 20}{\Delta L} - - - - - - - - (2)$$

We know that both the vessels are identical in shape and size and contain the same amount of ice but the conductivity of two vessels is different because they are made of different materials.

By comparing equations (1) and (2),

$$\frac{K_1 A\Delta T \times 25}{\Delta L} = \frac{K_2 A\Delta T \times 20}{\Delta L}$$

$$\frac{K_1}{K_2} = \frac{20}{25}$$

$$\frac{K_1}{K_2} = 0.8$$