CH # 10 MATTERS

Some important formulae:

$i)\ F = Kx \quad ii)\ Y = \frac{FL}{A\Delta L} \quad iii)\ \alpha = \frac{\Delta L}{L_1 \Delta T}$

iv) Area of wire = A = πr^2 v) Up thrust = ρVg (V = Ah)

10.1: A block of mass 0.1Kgg is attached to a spring and placed on horizontal frictionless table. The spring is stretched 20cm when a force of 5N is applied. Calculate the spring constant.

GIVEN:
Spring stretched = x = 20cm = 20/100 = 0.2m
Force applied = F = 5N

REQUIRED:
Spring constant = K = ?

SOLUTION:
F = Kx

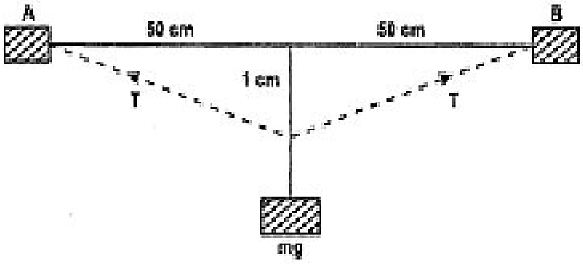
$$K = \frac{F}{x}$$

$$K = \frac{5}{0.2}$$

K = 25 N/m

10.2: A steel wire of diameter 0.8mm and length 1mm is clamped firmly at two points A and B which are 1m apart and in the same horizontal plane a block is hung from the middle point of the wire such that the middle point sages by 1cm. Calculate the mass of the block (Y = 2 x 10^{11} N/m^2 = 2 x 10^{12} Dyn/cm^2).

GIVEN:
Diameter of wire = D = 0.8mm



Radius of wire = r = 0.4mm = 0.4/10 = 0.04cm
Length of wire = L = 1m = 100cm
Distance between point A and B = d = 1m = 100cm
Sagging of the middle point = 1cm
Value of Young's modulus = Y = 2 x 10^{12} Dyn/cm^2

REQUIRED:
Mass of block = m = ?

SOLUTION:
According to Young's Modulus formula,

$$Y = \frac{FL}{A\Delta L}$$

$$F = \frac{YA\Delta L}{L}$$

$$mg = \frac{YA\Delta L}{L} \ (\because F = W = mg)$$

$$m = \frac{YA\Delta L}{Lg} ----------- (1)$$

Now we find Area of cross section 'A' and change in length 'ΔL',

<u>Area of cross section,</u>
A = πr^2
A = (3.14) $(0.04)^2$
A = 0.005024cm^2

<u>Change in length,</u>
By using Pythagoras theorem,
In ΔACD,
$|Hyp|^2 = |Base|^2 + |Perp|^2$

$$|Hyp| = \sqrt{|Base|^2 + |Perp|^2}$$

$$|AD| = \sqrt{|AC|^2 + |CD|^2}$$

$$|AD| = \sqrt{|50|^2 + |1|^2}$$

$$|AD| = \sqrt{2500 + 1}$$

$$|AD| = \sqrt{2501}$$

$$|AD| = 50.0099$$

For total length,
AB = 2 x 50.0099
AB = 100.0199cm
Now,
ΔL = AB – L
ΔL = 100.0199 – 100
ΔL = 0.0199cm
By putting these values in equation (1),

$$m = \frac{(2 \times 10^{12})(0.005024)(0.0199)}{(100)(980)}$$

$$m = \frac{199955200}{98000}$$

m = 2040.35gm or approx. 2Kg. (The Answer is wrong printed in book according to given data)

10.3: A steel wire 8m long and 4mm in diameter is fixed horizontally between two rigid supports. Calculate the increase in tension when the temperature falls by 10^0C (α for steel = $12 \times 10^{-6} K^{-1}$ and Y for steel = 2×10^{11} N/m^2).

GIVE:

Length of wire = L = 8m

Diameter of wire = D = 4mm

Radius of wire = r = 2mm = 2/1000 = 0.002m

Temperature decreased or changed = $\Delta T = 10^0C$

Co-efficient of linear expansion = $\alpha = 12 \times 10^{-6} K^{-1}$

Young's modulus for steel = $Y = 2 \times 10^{11}$ N/m^2

REQUIRED:

Increase in tension = F = ?

SOLUTION:

According to Young's modulus formula,

$$Y = \frac{FL}{A\Delta L}$$

Where F is the force of tension produce in wire,

$$F = \frac{YA\Delta L}{L} - - - - - - - - - (1)$$

Now we find Area of cross section 'A' and change in length 'ΔL',

<u>Area of cross section,</u>

$A = \pi r^2$

$A = (3.14)(0.002)^2$

$A = 0.00001256 m^2$

<u>Change in length,</u>

By using co-efficient of linear thermal expansion we have,

$\Delta L = \alpha L_1 \Delta T$

$\Delta L = (12 \times 10^{-6})(8)(10)$

$\Delta L = 0.00096$m

Using equation (1),

$$F = \frac{2 \times 10^{11} \times 0.00001256 \times 0.00096}{8}$$

$$F = \frac{2411.52}{8}$$

F = 301.44 N

10.4: A 2Kg object is suspended from a copper wire 2.5m long and 0.6mm in diameter. How much will the wire stretched due to gravitational force on the 2Kg object. Young's modulus for copper is 12.5×10^{10}N/m^2.

GIVEN:

Mass of object = m = 2Kg

Length of wire = L = 2.5m

Diameter of wire = d = 0.6mm

Radius of wire = r = 0.3mm = 0.3/1000 = 0.0003m

Young's modulus for copper= $Y = 12.5 \times 10^{10}$ N/m^2

REQUIRED:

Wire stretched due to gravitational force = Change in length = ΔL = ?

SOLUTION:

According to Young's modulus formula,

$$Y = \frac{FL}{A\Delta L}$$

$$\Delta L = \frac{FL}{YA}$$

$$\Delta L = \frac{mgL}{YA}(F = W = mg) - - - - - - - - - (1)$$

<u>Area of cross section,</u>

$A = \pi r^2$

$A = (3.14)(0.0003)^2$

$A = 0.000000282 m^2$

Using equation (1),

$$\Delta L = \frac{2 \times 9.8 \times 2.5}{12.5 \times 10^{10} \times 0.000000282}$$

$$\Delta L = \frac{49}{35250}$$

$\Delta L = 1.39 \times 10^{-3}$m or approx. 0.13cm

10.5: A block of mass 92Kg and volume 0.031m^3 lies at the bottom of the sea. How much force is needed to lift it. Take 1.03×10^3Kg/m^3 as the density of sea water.

GIVEN:

Mass of block = m = 92Kg

Volume of block= V = 0.031m^3

Density of water = $\rho = 1.03 \times 10^3$Kg/m^3

REQUIRED:

Force to lift the block = Apparent weight of block = ?

SOLUTION:

We know that,

Apparent weight = Actual weight – Up thrust - - - - - - - - - - - (1)

<u>Actual weight,</u>

Actual weight = mg

Actual weight = 92 x 9.8

Actual weight = 901.6N

<u>Up thrust,</u>

Up thrust = Ahρg

Up thrust = Vρg (Ah = V)

Up thrust = 0.031 x 1.03 x 10^3 x 9.8

Up thrust = 312.914N

Using equation (1),

Apparent weight = 901.6 – 312.94

Apparent weight = 588.686 or

Apparent weight = 590N

10.6: A canal lock gate is 10m wide and 10m deep. Calculate the thrust acting on it assuming that the water in the canal is in level with the top of the gate. Density of water is 1000Kg/m³.

GIVEN:

Width of gate = 10m

Depth of gate = h = 10m

Area of gate = A = 10m x 10m = $100m^2$

Density of water = ρ = $1000Kg/m^3$

REQUIRED:

Up thrust on gate = ?

SOLUTION:

Up thrust = Ahρg

Up thrust = 100 x 10 1000 x 9.8

Up thrust = 9.8 x 10^6N

10.7: A solid body floating on water has 1/5 of its volume above the surface of water. What fraction of its volume will project above the surface, if it floats in a liquid of specific gravity 1.60. Take density of water = 1gm/cm³.

GIVEN:

$\text{Volume of object above water} = \frac{1}{5}\text{V}$

$\text{Volume of object in water} = \frac{4}{5}\text{V}$

Density of water = ρ_1 = $1gm/cm^3$

Density of liquid = ρ_2 = $1.60gm/cm^3$

REQUIRED:

Volume fraction of a solid body above the surface of liquid = ?

SOLUTION:

Let,

W_1 = Weight of water displaced by the body.

W_2 = Weight of liquid displaced by the body.

X = Fraction of solid body volume above the liquid

W = Ahρg

W = Vρg ($\therefore$ Ah = V)

$W_1 = V\rho_1 g$

$W_1 = \frac{4}{5}\text{V x 1 x g}$

$W_1 = \frac{4}{5}\text{V g}$

Similarly,

$W_2 = VX\rho_2 g$

$W_2 = V \times X \times 1.60 \times g$

$W_2 = 1.60\ VXg$

According to Archimedes' principle,

$W_1 = W_2$

$\frac{4}{5}\text{V g} = 1.60\ VXg$

$X = \frac{4}{5 \times 1.6}$

$X = (0.5)\ or\ \frac{1}{2}$

10.8: A cork of specific gravity 0.15 floats in water (Sp. Gravity 1.025) with 10CC above the surface. Calculate the mass of cork. (Hint: CC stands for cm³)

GIVEN:

Specific gravity of water = ρ_1 = 1.025

Specific gravity of cork = ρ_2 = 0.15

Volume of cork above the surface of water = V = 10cc

REQUIRED:

Mass of cork = m = ?

SOLUTION:

Volume of cork under water = (V – 10)

Weight = $W_2 = V\rho_2 g$

Weight = W_2 = V x 0.15 x g

Weight = W_2 = 0.15Vg

Weight of displaced water = W_1 = V ρ_1g
Weight of displaced water = W_1 = (V – 10) x 1.025 x g

According to Archimedes principle or law of floatation,
$W_1 = W_2$
(V – 10) x 1.025 x g = 0.15Vg
(V – 10) x 1.025 = 0.15V
1.025V – 10.25 = 0.15V
1.025V – 0.15V = 10.25
0.875V = 10.25
$$V = \frac{10.25}{0.875}$$
V = 11.714 cc

We know that density is given by,
$$\rho_2 = \frac{m}{V}$$
$$m = V\rho_2$$
m = 11.714 x 0.15
m = 1.75gm

10.9: A body of density D is left free in a liquid of density d (D > d). Prove that the downward acceleration of the body while sinking in the liquid is given by the equation.

$$a = \left(1 - \frac{d}{D}\right)g$$

PROOF:
According to Newton's 2nd law of motion,
F = ma - - - - - - - - - - - - (1)
F = Apparent weight
Apparent weight = Actual weight – Up thrust
Apparent weight = mg – Vdg
Where F = Apparent weight = ma
ma = mg – Vdg
$$\mathrm{a} = \frac{\mathrm{mg}}{m} - \frac{\mathrm{Vdg}}{m}$$

$$\mathrm{a} = \mathrm{g} - \frac{\mathrm{Vdg}}{m}$$
$$\mathrm{a} = \left(1 - \frac{\mathrm{Vd}}{m}\right)g ------(2)$$
$$\text{Where } D = \frac{m}{V} \; or \; \frac{1}{D}$$
$$= \frac{V}{m} \; by \; putting \; it \; in \; above \; equation$$
$$\mathrm{a} = \left(1 - \frac{\mathrm{d}}{D}\right)g$$

10.10: A block of wood (Sp. Gravity 0.85) floats on water. Some kerosene oil of specific gravity 0.82 is poured on the surface of water until the wooden block is just immersed. Calculate the fraction of block lying below the surface of water in the second case.

SOLUTION:
Let the volume of water below water = V_1
And volume of block = V_2
Specific gravity of wood = 0.85
Density of kerosene oil = 6000Kg/m^3
Density of kerosene oil = 1000Kg/m^3
Weight of kerosene oil = $W_1 = V_1\rho_1 g$
Weight of kerosene oil = $W_1 = V_1$ x 6000 x g
Weight of kerosene oil = $W_1 = 6000V_1g$
Weight of water= $W_2 = V_2\rho_2 g$
Weight of Water = $W_2 = V_2$ x 1000 x g
Weight of water = $W_2 = 1000V_2g$
In case the body sinks in water and kerosene oil then,
$W_1 = W_2$
$6000V_1g = 1000V_2g$
$$V_1 = \frac{1000V_2}{6000}$$
$$V_1 = \frac{1}{6}V_2 \text{ is the volume of block below water}$$