CH # 07 CIRCULAR MOTION AND GRAVITATION

UNIFORM CIRCULAR MOTION:

"If a body moves along a circular path with a uniform speed then it is said to be in uniform circular motion."

ANGULAR DISPLACEMENT:

"The angle subtended at the centre of the circle is called angular displacement"

- It is represented by 'θ'.
- Unit of angular displacement is RADIAN.

$$1 \text{ Radian} = 57.3^0$$

CENTRIPETAL ACCELERATION:

"Acceleration of a body in a circular path is called centripetal acceleration."

FORMULA:

$$\text{Centripetal acceleration} = \frac{(\text{Speed})^2}{\text{Radius}}$$

$$\mathbf{ac = \frac{V^2}{r}}$$

- Centripetal acceleration represented by a_c.

Unit:

Unit of centripetal acceleration is m/s^2.

CENTRIPETAL FORCE:

"A centre seeking force which keeps the object moving in a circle is called centripetal force."

FORMULA:

$$Fc = ma_c$$

$$Fc = \frac{mV^2}{r}$$

- Centripetal force represented by Fc.

UNIT:

Unit of centripetal Force is Newton (N).

Example:

- Electron revolves around the nucleus of atom in circular orbit.
- Planets revolve around the sun.

FACTORS AFFECTING ON CENTRIPETAL FORCE:

- **Mass of object:**
  Centripetal force is directly proportional to the mass of the object.

- **Magnitude of square of velocity:**
  Centripetal force is directly proportional to the square of the magnitude of velocity.

- **Radius of the circle in which object moves.**

Centripetal force is inversely proportional to the radius of the circle in which object moves.

CENTRIFUGAL FORCE:

"The outward reacting force on the result of centripetal force is called centrifugal force."

- Centrifugal force is directed away from the centre of the circle.

EXAMPLES:

- Drying cloth in washing machine.
- Separation of cream from milk.
- Separation of sugar crystals from molasses.

CENTRIFUGE:

It is an appliance used to separate heavier particles from lighter particles in a liquid.

WORKING OF CENTRIFUGE:

The liquid is rotated in a cylinder vessel at high speed with the help of an electric motor. The heavier move away from the axis of rotation and lighter particles move near to the axis of rotation.

EXAMPLES:

1. **WASHING MACHINE DRYER:**
   In washing dryer the wet clothes are rotated at high speed. The water particles due to centrifugal force throw outward through the hole in the wall of outer vessel as the drum containing wet clothes rotates.

2. **CREAM SEPARATOR:**
   A cream separator is a type of centrifuge in which milk is rotated at high speed, the lighter cream particles collect near the axis while the skimmed milk moves away from axis of rotation.

3. **SUGAR CRYSTALS SEPARATOR:**
   Sugar crystals are separated from molasses with the help of centrifuge. When sugar solution is rotated in a cylindrical vessel, the sugar crystal move away from the axis of rotation and collected on the net around the vessel drum.

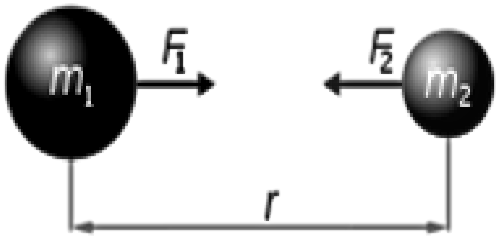
GRAVITATION:

"The study of force of attraction between bodies is called gravitation."

NEWTON'S LAW OF UNIVERSAL GRAVITATION:

Statement:

"Everybody in the universe attract every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres."



Explanation:

Let "m_1" and "m_2" be the masses of two spherical bodies, such that their centre are separated by a distance "r" , then according to

$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

the Newton's law of

Universal gravitation the force "F" with which they attract each other is given by:

$$F \alpha m_1 m_2 \_\_\_\_\_ (i$$

$$F \alpha \frac{1}{r^2} \quad \_\_\_\_\_ (ii$$

By combining eq (i &eq (ii we get:

$$F \alpha \frac{m_1 m_2}{r^2}$$

$$\boxed{F = G \frac{m_1 m_2}{r^2}}$$

*Where "G" is the universal gravitational constant and its value is:
G = 6.67 x $10^{-11}$$Nm^2/Kg^2$or G = 6.67 x $10^{-8}$$Dyn.cm^2/gm^2$.

APPLICATIONS OF NEWTON'S LAW OF UNIVERSAL GRAVITATION:

- Mass of earth.
- Variation of 'g' with altitude and depth.
- Orbital velocity.

MASS OF EARTH:

Consider a body of mass "m" lying on the surface of earth so that the distance between the centre of the earth and the centre of the body is equal to the radius "R" of the earth.

The force with which earth pulls the body towards its centre is known as its weight "W" given by:

$$W = mg$$

But according to the Newton's law of gravitation the force "F" with which earth pulls the body towards its centre given by:

$$F = G \frac{m M_e}{R^2}$$

Where "Me" is the mass of the earth and "F" & "W" represents the same force.

$$F = W = mg$$

$$mg = G \frac{m M_e}{R^2}$$

$$g = G \frac{M_e}{R^2}$$

$$\boxed{Me = \frac{gRe^2}{G}}$$

Where,

- G = 6.67 x 10^{-11} Nm^2/Kg^2
- g = 9.8 m/s^2
- R = 6.4 x 10^6 m

Substitute the value of G, g & R we get:

$$\mathrm{Me} = \frac{9.8 \text{ x } (6.4 \text{ x } 10^6)^2}{6.67 \text{ x } 10^{-11}}$$

$$\boxed{\text{Me} = 6 \text{ x } 10^{24} \text{ Kg}}$$

VARIATION OF g WITH ALTITUDE:

The acceleration of gravity "g" is the acceleration imparted to a body by its own weight. Consider a body of mass "m" on the earth's surface. The gravitational force of attraction between the body and earth is equal to the weight of body.

$$\mathrm{F = W = \frac{GmM_e}{R^2}}$$

$$\mathrm{mg = \frac{GmM_e}{R^2}}$$

The value of g at the surface of the earth will be,

$$\mathrm{g = \frac{GM_e}{R^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\mathrm{i})$$

If body is moved up to 'h'height from earth surface the acceleration due to gravity at that point is

$$\mathrm{g_h = \frac{GM_e}{(R+h)^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\mathrm{ii})$$

Divide equation (i) by (ii), we get

$$\frac{g}{g_h} = \frac{GM_e}{R^2} \div \frac{GM_e}{(R+h)^2}$$

$$\frac{g}{g_h} = \frac{GM_e}{R^2} \times \frac{(R+h)^2}{GM_e}$$

$$\frac{g}{g_h} = \frac{(R+h)^2}{R^2}$$

$$\frac{g}{g_h} = \frac{R^2 + 2Rh + h^2}{R^2}$$

$$\frac{g_h}{g} = \frac{R^2}{R^2} + \frac{2Rh}{R^2} + \frac{h^2}{R^2}$$

$$\frac{g_h}{g} = 1 + \frac{2h}{R} + \frac{h^2}{R^2}$$

As $\frac{h^2}{R^2}$ is negligible small

$$\frac{g_h}{g} = \left(1 + \frac{2h}{R}\right)$$

$$\frac{g}{g_h} = \frac{1}{\left(1+\frac{2h}{R}\right)}$$

$$\frac{g}{g_h} = \left(1+\frac{2h}{R}\right)^{-1}$$

By the binomial expansion,

$$\frac{g}{g_h} = \left(1-\frac{2h}{R}\right)$$

$$\boxed{g = g_h\left(1-\frac{2h}{R}\right)}$$

This equation shows that value of g decreases as we increase in height.

ARTIFICIAL SATELLITE:

"The objects which are used for communication and space research, revolve around the earth under the force of gravity, which provide the necessary centripetal force, made by human not nature is called artificial satellite."

ORBITAL VELOCITY:

"The velocity with which the satellite moves around the earth is called orbital velocity."

Derivation:

Consider a satellite of mass "m" moving in an orbit of radius "r" with velocity "V" around the earth. The gravitational force of attraction between the satellite and the earth provide necessary centripetal force.

If Me is the mass of the earth, than

$$\text{Gravitational force} = \frac{GM_em}{r^2} \text{ and}$$

$$\text{Centripetal force} = \frac{mV^2}{r}$$

$$\frac{GM_em}{r^2} = \frac{mV^2}{r}$$

Or

$$V^2 = \frac{GM_e}{r} - - - - - - - - - - (i)$$

If 'g' is the acceleration due to gravity at the position of satellite then

$$mg = \frac{GM_em}{r^2}$$

$$gr = \frac{GM_e}{r} - - - - - - - - - - (ii)$$

Comparing equation (i) and (ii) we have

$$\boxed{V^2 = gr \text{ or } V = \sqrt{gr}}$$

If satellite revolving near to the surface of the earth then r = Re = radius of earth and the value of orbital velocity becomes,

$$V = \sqrt{gRe}$$

From astronomical data,
g = 9.8m/s^2 and Re = 6.4 x 10^{6}m

$$V = \sqrt{(9.8 m/s^2)(6.4 \times 10^6 m)}$$

V = 7920 m/s

OR

V = 7.920 Km/s ≈ 8 Km/s